Logic Will Take You By the Throat

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"Logic will take you by the throat and force you" exclaimed the exasperated Achilles to the mischievous turtle in Lewis Carrol's famous parable. In this talk, I explore how Logic seemingly forces us to take positions at odds with currently almost universally accepted beliefs.

In 1933, Alonzo Church became alarmed about the new science of computation that he was developing with colleagues and so published the following concern: "Indeed, if there is no formalization of logic as a whole, then there is no exact description of what logic is, for it in the very nature of an exact description that it implies a formalization. And if there no exact description of logic, then there is no sound basis for supposing that there is such a thing as logic." The crucial nature of the paradox which Church had discovered was missed for over eight decades until pressing needs of Computer Science brought it to the forefront.

Mathematics here means the common foundation of all classical mathematical theories from Euclid to the mathematics used to prove Fermat's Last. Direct Logic provides categorical axiomatizations of the Natural Numbers, Real Numbers, Ordinal Numbers, Set Theory, and the Lambda Calculus meaning that up to a unique isomorphism there is only one model that satisfies axioms of the respective theories. (It is important that the isomorphism be unique in order to precisely specify a mathematical structure that is axiomatized.) Strong types block all known paradoxes including Berry, Burali-Forti, Girard, Russell, etc.

This talk shows how strong types for Mathematics can achieve the following:

- Mathematics self proves that it is "open" in the sense that theorems are not computationally enumerable by a provably total procedure. However, correctness of proofs is computationally decidable.
- Mathematics self proves that it is formally consistent in the sense it proves that there is no proposition that is both provable and disprovable. However, this does not per se prove constructive consistency meaning that no contradiction can be derived from stated axioms and rules of inference.
- Strong mathematical theories for Natural Numbers, Ordinals, Set Theory, the Lambda Calculus, Actors, etc. are inferentially decidable, meaning that every true proposition is provable and every proposition is either provable or disprovable. However, theorems of these theories are not enumerable by a provably total procedure.

Of course, the above results are completely at odds with 1st order logic. Because of strong types for Mathematics, Gödel's famous proposition "I'm unprovable." cannot be constructed, which is good because Wittgenstein showed that the proposition leads to inconsistency in Mathematics. Also, adding a true proposition to a 1st order theory can make it inconsistent. Attempting to 1st order axiomatize the natural numbers or concurrent computation systems fail disastrously because it is computationally undecidable if adding a given true proposition renders the axiomatization inconsistent.

Consistent strong mathematical theories can be freely used without introducing *additional* inconsistent information into inconsistency robust empirical theories that will be the core of future Intelligent Applications.

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